Theories of Ordinal Marginal Utility

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Abstract
As proved by Hicks and Allen (1934a, 1934b), the law of diminishing marginal utility, defined as the negative sign of the second partial derivative of the utility function, is incompatible with ordinalism. Yet, there were some economists, who attempted to redefine the law within ordinalist framework. This essay reviews the attempts of Mayston (1976), Bernardelli (1938, 1952), Rothbard (1977, 2004) and McCulloch (1977). It concludes that the law can be made compatible with ordinal concept of utility. Nevertheless, an inconsistency in Rothbard's treatment is discovered: the law of marginal utility is found irreconcilable with the concept of demonstrated preference. Bernardelli's approach, on the other hand, can be appealing even for present economists, because it can incorporate various forms of status quo bias.

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Since the Hicks-Allen (1934a, 1934b) reformulation of demand theory, the law of diminishing marginal utility (LDMU) has been in disfavour with most economists. The reason is its alleged incompatibility with the ordinal concept of utility. Yet, there were economists who did not want to give up the once-famous law easily; consequently, there were multiple different attempts to rescue the law. These attempts proceeded along the following two main lines:

(i) It was admitted that ordinalism and LDMU were incompatible; however, it was argued that it is ordinalism, not LDMU, what must give way and that utility function is in fact determinate (Robertson (1952, 1954), Armstrong (1939, 1955)).[1]

(ii) It was argued that ordinalism and LDMU were compatible. The arguments involved a reformulation of LDMU and were of two types: first type of argument accepted Hicks-Allen approach and redefined LDMU within their framework (Mayston 1976); second type drew from the older German-Austrian tradition of ordinalism, which has roots in the works of Andreas Heinrich Voigt, Oskar Kraus and František (Franz) Čuhel,[2] and attempted to create a viable alternative to Hicks-Allen approach. It was especially Čuhel who – thorugh the influence on Ludwig von Mises – made the concept of ordinal marginal utility one of the basic tenets of the Austrian school (Mises 1990, 2003, 1953, 1981:9; Rothbard 1998, 1997:12). The Austrian and other Mises-inspired approaches are not homogeneous and must be considered separately. At least those of Bernardelli, Rothbard and McCulloch can be distinguished.

This paper – as its title suggests – focuses on (ii), i.e. the attempts to reconcile ordinalism and LDMU. Its purpose is to examine, whether there really can be an ordinal marginal utility or whether it is a chimera and ordinal marginal utility is a result of deficiency in the theory. It arrives at the conclusion that there *can* be ordinal marginal utility; however, it argues that the best-known version of the theory – the Austrian (Mises-Rothbard's) version, which is influential until present – is unsatisfactory for it leaves important questions unanswered. Bernardelli's approach, on

[1] Somewhat special case is Kennedy (1954), who accepted determinateness of utility function, yet dismissed LDMU.

[2] For more on Voigt's contributions see Schmidt and Weber (2008). Basic facts about the life and work of Čuhel can be found in Hudík (2007). High and Bloch (1989) review the works of the authors writing in this tradition in the first quarter of the 20th century. These authors include, besides those already mentioned, Leo Schoenfeld, Oskar Englaender, Oskar Morgenstern, Paul Rosenstein-Rodan and Alexander Bilimovič.

the other hand, if appropriately interpreted can be appealing even to the present theoreticians because it can incorporate insights from behavioural economics.

The paper proceeds as follows: Part 1 briefly reviews Hicks-Allen reformulation of utility theory, in particular, their argument against the LDMU. Part 2 presents Mayston's attempt to rescue LDMU within the Hicks-Allen framework. The next parts deal with the Austrian or Austrian-inspired authors: Bernardelli's, Rothbard's and McCulloch's approaches are examined in Parts 3, 4 and 6, respectively. A brief comment on High's and Bloch's interpretation of the Austrian theory is inserted as part 5. Part 7 concludes.

1. From marginal utility to marginal rate of substitution

The well-known Hicks' and Allen's argument against LDMU can be summarized as follows: Let $u(x)$ be a utility function; ordinalism requires that $u(x)$ is unique up to positive monotonic transformation. Formally, for any transformation $T[u(x)]$, it must be that $T' > 0$. Assuming that LDMU is defined through the sign of the second partial derivative as $u_{ii} < 0$ (an important assumption as we shall see), it is straightforward to show that this sign is under positive monotonic transformations indeterminate:

$$\frac{dT'u_i}{dx_i} = T'u_{ii} + T''u_i u_i \, , \tag{1}$$

which has always the same sign as u_{ii} only if $T''u_i u_i = 0$. Fortunately, ordinal concept of utility enables marginal utilities to be compared at one particular point and thus a measure can be defined, which remains invariant under positive monotonic transformations. This measure is marginal rate of substitution (MRS) and is defined as a ratio of marginal utilities at one point:

$$MRS_{ji} = \frac{u_i(x)}{u_j(x)} \tag{2}$$

Thus the LDMU can be replaced by the law of diminishing marginal substitution. Since the above argument is obviously correct, the LDMU can be saved only by defining LDMU in other way than $u_{ii} < 0$ (given that we do not want to violate ordinalism). As we shall see, Mayston defined it through the sign of total differential and other authors attempted to construct different utility functions.

2. Maytson's reformulation of LDMU

Mayston (1976) observed that MRS is not the only measure that could be used under positive monotonic transformations of $u(x)$; moreover, he argued that LDMU can be formulated if appropriate interpretation is adopted. He pointed out that marginal utilities can be compared not only at one particular point but also between points on the same indifference surface. Consider a transformation $T[u(x)]$,, such that $T' > 0$:

$$\frac{T'[u(x')]u_i(x')}{T'[u(x'')]u_j(x'')} = \frac{u_i(x')}{u_j(x'')} \tag{3}$$

The equality holds if $u(x') = u(x'')$, i.e. if x and y lie on the same indifference surface.[3] LDMU can then be rescued if it is formulated along indifference surface. It can then be defined through the negative sign of the *total* differential:

$$\frac{du_i}{dx_i} = u_{ii} + u_{i2}\frac{dx_2}{dx_i} + \ldots + u_{in}\frac{dx_n}{dx_i} \tag{4}$$

$$\frac{d(T'u_i)}{dx_i} = T''\left(u_i + u_2\frac{dx_2}{dx_i} + \ldots + u_n\frac{dx_n}{dx_i}\right)u_i + \\ +T'\left(u_{ii} + u_{i2}\frac{dx_2}{dx_i} + \ldots + u_{in}\frac{dx_n}{dx_i}\right) \tag{5}$$

Since the LDMU is considered along an indifference surface, we must have

$$u_1 + u_2\frac{dx_2}{dx_1} \ldots + u_n\frac{dx_n}{dx_1} = 0 \tag{6}$$

and therefore

$$\frac{d(T'u_i)}{dx_i} = T'\left(u_{ii} + u_{i2}\frac{dx_2}{dx_i} + \ldots + u_{in}\frac{dx_n}{dx_i}\right) \tag{7}$$

Since $T' > 0$, $d(T'u_i)/dx_i$ has the same sign as du_i/dx_i. Mayston also argues that this interpretation of LDMU can be found in Marshall and

[3] MRS is of course a special case of (3) for $x' = x''$.

gives some evidence in favour of this hypothesis. Nonetheless, it is doubtful whether this interpretation is in line with our introspective understanding of the law; and it is precisely this understanding, easily accessible to everyone, which was the main argument for retaining the LDMU.[4]

3. Bernardelli's attempt to rescue LDMU

We now examine the approaches that are inspired by German-writing ordinalists, especially by Mises (1953, 2003, 1996). These approaches have one thing in common: they think of marginal utility as a *utility of a change*. rather than a *change in (total) utility*. Consequently the difference between the between *total* and *marginal* utility is denied;[5] total utility is considered to be nothing but a marginal utility of a larger unit and consequently, there is only one "species" of utility function that is both total and marginal. Nonetheless, except for this common characteristic the group is heterogeneous: some (Rothbard) present non-formalized version of utility theory and found the mathematical language unsuitable for treating the matter. Others (Bernardelli and McCulloch) try to translate the literary arguments of earlier economists into formal language. We shall stick to the chronological order and start with the approach coined by Bernardelli (1938, 1952). His approach will also serve as a kind of benchmark to which we relate other authors in this tradition.

Instead of starting with the utility function $u(x)$ and then derive marginal utility function from it, Bernardelli's idea was to start with marginal utility function right away.[6] This function he defined as $f(x_1^0,...,x_n^0;h_1,...,h_n)$, where x_i^0 is a stock of the good i in possession and h_i is an increment of the good i. Total utility can then be represented as $f(0,...,0;h_1,...,h_n)$.[7] The laws of utility can be defined as

$$\frac{\partial f(x^0;h)}{\partial h_i} > 0 \text{ for all values of } x^0, \qquad (8)$$

expressing that a greater stock of a commodity is preferred to a smaller one; and

[4] Cf. Bernardelli (1952:268), Arrow (1951:529).

[5] With the possible exception of McCulloch (1977) – but even he derives the total and marginal utility function from the "utility" of wants.

[6] Bernardelli (1938:201n) finds this idea already in Pigou (1935).

[7] In the 1952 paper Bernardelli dismissed the assumption $x^0 = 0$, arguing that this would mean that a person is placed in a position of naked void, which either does not make sense or is irrelevant for market behaviour.

$$\frac{\partial f(x^0;h)}{\partial x_i^0} < 0\,, \tag{9}$$

expressing the LDMU. It is straightforward to see that the function *f* is unique up to positive monotonic transformations since all the properties are defined through the signs of first derivatives.

There does not seem to be a problem with Bernardelli's approach; yet, due to some awkwardness of his exposition, it was misunderstood. The difficulty lied in the fact was that Bernardelli gave the impression that he derived his *f* function from a "total utility" function *u* as follows:[8]

$$f(x_1^0,...,x_n^0;h_1,...,h_n) \equiv u(x_1^0+h_1,...,x_n^0+h_n) - u(x_1,...,x_n) \tag{10}$$

He then tried to establish the equivalence of u_{ii} and $\partial f(x^0;h)/\partial x_i^0$. Consequently, he became an easy prey of critics: Samuleson (1939) proved that if (10) is the case, then the proposed formalization of LDMU is not invariant under positive monotonic transformations of *u* (sic!), which is according to Samuelson what really matters; Lancaster (1953) correctly shows that the equivalence of u_{ii} and $\partial f(x^0;h)/\partial x_i^0$ cannot be generally established. Bernardelli's solution then, in Samuelson's words, leaves matters just where they were.[9]

A minor change can however remedy the problem: given that $h_i = x_i^1 - x_i^0$, where x_i^1 is a new quantity of commodity, we can write

$$f(x^0;h) \equiv g(x^0,x^1) \tag{11}$$

Now we can think of consumer as having preferences not over *states* (as it is the case with the *u* function), but over the *changes of states*. It now can be shown that if marginal utility is understood as *utility of a change*, Bernardelli's argument becomes free from the faults traced by Samuelson and Lancaster.

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[8] In his 1952 article he actually does derive *f* from *u*, but treats *u* only as an "auxiliary function".

[9] If my interpretation of Bernardelli is correct (cf. Bernardelli's (1939, 1954) replies to Samuelson and Lancester), then Armstrong (1955) and Kennedy (1954) misunderstood Bernardelli's argument in the same way as Samuleson and Lancaster.

As an illustration, consider the following statement: "Marginal utility of the 3rd unit of commodity 1 is greater than marginal utility of the 5th unit of the same commodity," which can be translated into the statement "I prefer the 3rd unit of the commodity 1 to the 2nd unit of the commodity 1 more than I prefer the 4th unit to 5th unit." This statement can mean two things: standard interpretation is that the difference between $u(3, x_2 ..., x_n)$ and $u(2, x_2 ..., x_n)$ is greater than the difference between $u(5, x_2 ..., x_n)$ and $u(4, x_2 ..., x_n)$, i.e.

$$u(3, x_2 ..., x_n) - u(2, x_2 ..., x_n) > u(5, x_2 ..., x_n) - u(4, x_2 ..., x_n) . \qquad (12)$$

Another interpretation would be: "I prefer the change from 2 units to 3 units to the change from 4 units to 5 units", i.e.

$$g(2, x_2^0 ..., x_n^0; 3, x_2^0 ..., x_n^0) > g(4, x_2^0 ..., x_n^0; 5, x_2^0 ..., x_n^0) \qquad (13)$$

Naturally, the questions arises, under what conditions are (12) and (13) equivalent. Interestingly, it was Samuelson (1938), who gave the answer: he proved that the equivalence holds under somewhat restrictive condition, namely if

$$g(x^0, x^1) + g(x^1, x^2) = g(x^0, x^2) \text{, for all } x^0, x^1, x^2 \qquad (14)$$

He also noted that if the initial state x^0 is fixed, then the function g can serve as a function representing preferences over (end) states. To put it differently, u can be viewed as a special case of g, for the cases where initial states are fixed. Why do we need u then? Why cannot we base the consumer theory on g? To these questions Samuelson did not give an answer. It is unfortunate for Bernardelli, that he did not relate his ideas to this earlier Samuelson's paper and took the equivalence of (12) and (13) for granted and attempted to prove the equivalence of u_{ii} and $\partial f(x^0; h) / \partial x_i^0$. He should have disposed with the function u altogether and realize that his definition of marginal utility is in fact different from the standard one.

It now remains to show that our reinterpretation of Bernardelli's argument as suggested above is in line with what he wrote in the texts but failed to give it a satisfactory formal expression. First note that as far as the 1938 article is concerned, the equivalence of of u_{ii} and $\partial f(x^0; h) / \partial x_i^0$ is only discussed in Appendix, not in the main text. In his reply to Samuelson Bernardelli (1939) makes clear that in the Appendix he tried to explain how his function *f* can be linked up with the traditional utility function *u*. He also writes:

> "Now the very essence of my article, it will be recollected, was to demonstrate that such a representation of marginal utility [by the equation (10)] is inadmissible. All the arguments in the article (as distinct from the appendix) were centred round the thesis that it is this and only this conception of marginal utility as a difference of total utilities which causes the muddle of the measurability dispute." (Bernardelli 1939:88)

In the 1952 article the equation (10) made it to the main text; nonetheless, the function *u* is treated as an "auxillary function" – it is "merely an analytical device and need not be mentioned at all." (Bernardelli 1952:256).

Finally, it may be pointed out that Bernardelli's solution is attractive for present readers because it can account for the endowment effect, status quo bias and loss aversion; Bernardelli (1952:258) was aware of the fact (if not even motivated by it) that consumer choice depends on circumstances under which the choice is made. His approach thus not only rehabilitates the LDMU by generalizing standard theory of consumer behaviour but also solves some of the problems raised by behavioural economists.

4. Rothbard's approach

Rothbard's treatment of ordinal marginal utility is somewhat difficult to interpret and at a closer examination seems unsound. Our interpretation is that Rothbard's concept of ordinal marginal utility equals to Bernardelli's minus mathematical formalization plus the concept of demonstrated preference. It is convenient to pin down the crucial points of his argument and discuss them in turn. These points are: ordinality of utility, impossibility of mathematical treatment of utility, non-existence of total utility and the concept of demonstrated preference.

Ordinalism is of course common to all the authors mentioned; however, there is a special reason why we have to give it an extra discussion in Rothbard's case. This reason is that he attaches different meanings to the terms "ordinal", "cardinal" and "measurable" than other authors. When Rothbard speaks of measurability (or, equivalently, cardinality) he seem to have in mind a possibility of a transformation of subject-object relations to object-object relations.[10] For instance, in the case of temperature, it is a substitution of the statements like "(I feel that) the water is hot" (subject-object) for the statements like "the temperature of the water corresponds

[10] "Psychological magnitudes cannot be measured since there is no objectively extensive unit—a necessary requisite of measurement"(Rothbard 1977:10).

to such and such height of mercury column" (object-object). Ordinality is than to him simply non-measurability. For other economists, ordinality is uniqueness of utility function up to positive monotonic transformation, whereas cardinality (or measurability)[11] is uniqueness of utility function up to affine transformations. This divergent understanding of the terms causes permanent misunderstandings between neoclassical and Austrian versions of the consumer theory.

From Rothbard's concept of ordinality follows his argument of impossibility to treat utility mathematically:

> „If utilities can be subjected to the arithmetical operation of subtraction, and can be differentiated and integrated, then obviously the concept of marginal utility must imply cardinally measurable utilities" (Rothbard 1977:10).

It is indeed the case that cardinal utility functions are used by non-Austrian (Hicksian) ordinalists – but merely as *representations* of ordinal preferences. One can choose infinity of cardinal functions to represent one preference ordering and any such function is perfectly suitable for the purposes of the demand theory. Now if we use numbers to represent utilities and subject those to mathematical operations, it does not mean that we "measure" utility with these numbers – no interpretation is given to these numbers. To use an analogy, assume that instead of representing preferences with numbers we would represent them with letters in such a way that the most preferred alternative is assigned A, the second best B, etc. Would Rothbard argue that this is unjustifiable since utility cannot be "read"? The main reason why Rothbard's argument fails, however, is that one does not have to represent preference ordering with a utility function at all and work with preference relation (or choice function) instead.

We now move to what seems to be the substantial differences between the Austrian and non-Austrian treatment of utility; namely, dismissal of total utility and the concept of demonstrated preference. As for the first point, according to Rothbard there is no such thing as total utility: there is

[11] The ordinality-cardinality distinction under the above mentioned meaning is believed to originate with Hicks and Allen (1934a, 1934b) (Chipman 1960:215-216). Nonetheless, Schmidt and Weber (2008) argue that Hicks and Allen borrowed the ordinal-cardinal distinction from Edgeworth who in turn had learned it from Voigt. The identification of cardinality and measurability is attributable to Lange (1934); it was only in the context of discussions of von Neumann-Morgenstern utility function, when economists realized that this identification of cardinality and measurement was wrong and that the two are actually different concepts (Lewin 1996:1308n).

only marginal utility of a unit of a larger size. This part of Rothbard's argument is a bit unclear but probably he follows Bernardelli's approach (Rothbard 2004:314n): he explicitly writes that the "word 'marginal' presupposes *not* increments of utility, *but the utility of increments of goods*" (Rothbard 1977:12). This would mean that he implicitly adopts something like Bernardelli's utility function and has in mind equations (8) and (9) as representing the two laws of utility. Now if this is the case then he cannot use the concept of demonstrated preference, as will be presently shown.

According to Rothbard, demonstrated preference means that "actual choice reveals, or demonstrates, a man's preferences; that is, that his preferences are deducible from what he has chosen in action." (Rothbard 1977:2) Now the problem is that utilities of increments of goods under different circumstances cannot be demonstrated by choice. For example, it is impossible to demonstrate greater marginal utility of a change from the 2nd to the 3rd unit of a good than of a change from the 4th to the 5th unit (as represented by the equation (13) above). The only change that an individual can demonstrate, is a change from a given situation in which he founds himself. Hence, it does not make sense to consider his utility as a function of the initial state, because there is only one initial state possible in which he can choose. It follows that LDMU cannot be demonstrated by choice.

There seem to be no way how to reconcile the concept of marginal utility as a utility of change and at the same time the concept of demonstrated preference. One has to choose, whether to adopt the concept of marginal utility and make the theory "psychological" in the sense that it works not only with real but also with hypothetical choices, or to adopt demonstrated preference and give up marginal utility. Unfortunately, Rothbard and his followers do not address the problem and leave it unsolved.

5. High and Bloch's interpretation of the Austrians

Above we suggested that Rothbard's concept of ordinal utility is mutatis mutandis the same as Bernardelli's. High and Bloch (1989:358) offer a different interpretation of the Austrian theory of ordinal marginal utility; it will be argued that their interpretation is problematic.

The authors attempted to give the following formal proof of the marginal utility: Let $X = \{x_1, ..., x_n\}$, where x_i are different sizes of a stock of a good x and $x_i < x_j \Leftrightarrow i < j$. Define marginal utility function $\psi : X \to \mathbb{R}_+$ such that

$$\psi(x_i) > \psi(x_j) \Leftrightarrow x_i < x_j \tag{15}$$

Then it is straightforward to see that ψ is unique up to positive monotonic transformations.

There is nothing wrong with the proof. However, High and Bloch do not realize that they have to define *another* utility function that would account for the fact that higher supply of a good is preferred to a smaller supply thereof (cf. Rothbard 2004:25). This utility function would thus be defined as $\varphi: X \to \mathbb{R}_+$ such that

$$\varphi(x_i) > \varphi(x_j) \Leftrightarrow x_i > x_j \tag{16}$$

Then the relationship between these two functions would have to be formulated. If ψ is the marginal utility is φ cannot be marginal utility too. Is it φ the total utility? That would be certainly unacceptable to Rothbard and presumably to other Austrians too. Another objection would be that ψ does not account for complementarities among goods since it depends on the quantity of one good only.

6. McCulloch's theory of marginal use

McCulloch follows Menger (1950) in assuming that preference relation is not defined on the set of goods bundles but on the set of wants, or more precisely, on the set of different combinations of wants (*W**): if *W* is the set of wants then *W** is the set of all the subsets of *W*, i.e. $W^* = \{P \subset W\}$. Let $v: W^* \to \mathbb{R}$ be an ordinal utility function. McCulloch then considers how different bundles are valued by a consumer according to which combinations of wants can these bundles satisfy. In order to be able to compare his analysis with those considered before we may rewrite it as follows:[12] let *X* be the set of all commodity bundles; we may than define a "use" (or "production") function $s: X \to W^*$. We then have what may be called "total indirect utility function" (McCulloch does not employ this term):

$$v(w^*) \equiv v(s(x)) \tag{17}$$

Likewise, we may define an "indirect marginal utility function". First define "marginal use" function $m: X \times X \to W^*$; the interpretation is as

[12] We focus only on the possibility of ordinal marginal utility; other aspects of McCulloch's analysis are omitted.

follows: to each member $(x^0, x^1) \in X \times X$, where – as in the section 3 – x^0 represents an initial position and x^1 an end position, a member of *W** is assigned representing the additional wants that a consumer can satisfy as he moves from x^0 to x^1. For the indirect marginal utility function we then can write

$$v(w^*) \equiv v(m(x^0, x^1)) \tag{18}$$

It is clear that the function *s* is superfluous once the function *m* is defined; if McCulloch realized this, he would have been able to dispose with total utility – marginal utility distinction and – as did Bernardelli – work with marginal utility only. This, however, is a cosmetic detail compared to the next problem. McCulloch gets into difficulties when considering the case of complementary goods. The reason is that the function *m* assumes that as consumer increases amounts of goods, he satisfies an additional want without sacrificing any of those wants that he satisfied before the increase. It does not account for the possibility, typical for complementary goods, that sometimes as the amount of a good is increased a consumer substitutes lower ranked wants for higher ranked wants. To remedy this problem, McCulloch in effect argues that in such cases (that are far from exceptional!) the preference ordering has to be defined on a set $W^{**} = \{(P, S) \in W^* \times W^*\}$, where P are wants that are satisfied in the initial state and S are wants satisfied in the end state. The function of marginal use then has to be redefined as $\mu : X \times X \rightarrow W^{**}$ [13]

We now compare McCulloch's treatment with Bernardelli's. Apart from the distinction between total and marginal utility, which McCulloch preserves and Bernardelli abandons, the crucial difference between them seems to reduce to the problem, whether to assume preference ordering on the set *W** or directly on the set $X \times X$. Roughly speaking, the difference is that although both can formulate LDMU without violating ordinality assumption, Bernardelli has to *assume* it McCulloch can *derive* it from the preferences about wants.[14] McCulloch believes that the latter approach makes it possible to establish the LDMU as a consequence of more primitive notions rather than simply assume it. He, however, has to

[13] This is for us convenient translation of McCulloch's solution. He works with ordered pairs $(a, b) \subset W \times W$, where the first entry *a* represents the additional want satisfied and the second entry *b* represents the want whose satisfaction is sacrificed.

[14] Further restriction on the preference ordering must be imposed, namely, non-complementarity of wants or – as McCulloch calls it – unrelatedness: A set *W** is unrelatedly ordered if for any $Q, R \subset W$ we have $Q \succ R \Leftrightarrow Q - R \succ R - Q$ (McCulloch 1977:253). Reader will confirm that this property is rather implausible.

adopt some additional restrictions on preferences that make this deduction possible. It is beyond the scope of the present paper to decide, which approach is more convenient.

7. Conclusion

We have examined the possibility of ordinal marginal utility. The conclusion is that LDMU *can* be defined without violating ordinality of utility, in fact, there are at least two ways how to do it: one, adopted by Mayston (1976) remains within Hicks-Allen framework and thinks of marginal utility as total rather than partial derivative. The other solution defines marginal utility as a utility of a change rather than a change in total utility. This solution is (with some variations) adopted by Bernardelli (1938, 1952), Rothbard (1977, 2004) and McCulloch (1977). It would be interesting to examine, whether this second approach cannot be pursued further and form an alternative to the still prevalent Hick-Allen-Samuelson approach.

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